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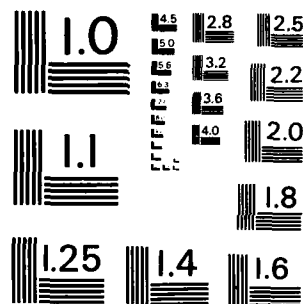
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September 1982

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DIGITAL IMAGES

George M. Reed

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ABSTRACT

This is a continuation of a series of papers on the digital geometry of three-dimensional digital images. In earlier reports, D. Morgenthaler and A. Rosenfeld gave symmetric definitions for simple surface points under the concepts of 6-connectivity and 26-connectivity, and they non-trivially characterized a simple closed surface (i.e., a subset of the image which separates its complement into an "inside" and an "outside") as a connected collection of "orientable" simple surface points. Later, the author and A. Rosenfeld established that the computationally costly assumption of orientability is unnecessary for 6-connectivity by proving that orientability, a local property, is implicitly guaranteed within the  $(3 \times 3 \times 3)$ -neighborhood definition of a 6-connected simple surface point. However, they also showed that no such guarantee exists for 26-connectivity. In this report, the author completes this investigation of simple closed surfaces by showing that orientability is ensured globally by 26-connectivity. Hence, a simple closed surface may be efficiently characterized as a connected collection of simple surface points regardless of the type of connectivity in consideration. ✓

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## 1. Introduction

Geometrical and topological characterizations of subsets of digital pictures play an important role in computer image analysis and pattern recognition [1]. Topological concepts such as connectedness and simple closed curves are well-understood in two-dimensional arrays [2] and have proven to be useful tools for a wide variety of image analysis tasks such as object extraction, thinning, and skeletonization.

With the increased interest in computed tomography and the three-dimensional representation of microscopic cross-sections and time sequences of images, it has now become desirable to develop a consistent and efficient theory for the understanding of geometric and topological properties of subsets of three-dimensional digital arrays. Early work in this area was done by Gray [3] and Park [4], and other authors [5,6] have considered generalizations of specific two-dimensional results to higher dimensions. However, the current series of papers on three-dimensional digital geometry written at the Computer Vision Laboratory of the University of Maryland ([7],[5],[9],[10],[11],[12],[13]) appears to be the first systematic study designed to develop the desired theory. This report is the latest in this series, and it completes the characterization of simple closed surfaces.

Simple surface points and simple closed surfaces were introduced in [10] to establish the three-dimensional analog of the two-dimensional Jordan Curve Theorem. The goal was to

define computationally efficient properties such that a subset of the 3D-lattice satisfying those properties separated the complement of that subset into an "inside" and an "outside." Indeed, this goal was to a large extent achieved in [10] where it was shown that a connected collection of "orientable" simple surface points provided the desired characterization. Unresolved, however, was the necessity of orientability, a property that unlike the other components of the characterization required the computationally costly examination of the  $(5 \times 5 \times 5)$  neighborhoods of the points under consideration. In [13], it was shown that orientability was actually guaranteed by the  $(3 \times 3 \times 3)$ -neighborhood restrictions imposed locally by simple surface points under 6-connectivity (one of the two types of connectedness to be considered). However, it was also shown in [13] that no such guarantee was imposed locally under 26-connectivity. In this paper, we succeed in showing that orientability is imposed globally by a 26-connected set of simple surface points. Hence, the desired characterization, which requires only the examination of  $(3 \times 3 \times 3)$ -neighborhoods, is established under both types of connectivity.

The approach to the characterization of surface properties in this paper and others in this series where surfaces are considered to be sets of voxels should be contrasted with that of Artzy, Frieder, and Herman [14] and Herman and Webster [15] in which surfaces are considered to be faces of voxels. The two approaches are complementary.

## 2. Connectivity and simple closed surfaces

Let  $\Sigma$  denote a 3D array of lattice points, which, without loss of generality, we may assume to be defined by integer valued triples of Cartesian coordinates  $(x,y,z)$ . We consider two types of neighbors of a point  $p = (x_p, y_p, z_p) \in \Sigma$ :

- (i) the neighbors  $(u,v,w)$  such that  $|x_p - u| + |y_p - v| + |z_p - w| = 1$
- (ii) the neighbors  $(u,v,w)$  such that  $\max\{|x_p - u|, |y_p - v|, |z_p - w|\} = 1$

We refer to the neighbors of type (i) as 6-neighbors of  $p$  (the face neighbors) and to the neighbors of type (ii) as 26-neighbors of  $p$  (the face, edge, and corner neighbors). The 6-neighbors are said to be 6-adjacent to  $p$ , and the 26-neighbors are said to be 26-adjacent to  $p$ . The statement that  $\alpha$  is a path from point  $p$  to point  $q$  in  $\Sigma$  means that there exists a positive integer  $n$  such that  $\alpha = \{p_0, p_1, \dots, p_n\} \subseteq \Sigma$  where  $p_0 = p$ ,  $p_n = q$  and  $p_i$  is adjacent to  $p_{i-1}$  for  $1 \leq i \leq n$ . The terms 6-path and 26-path are utilized depending on the type of adjacency under consideration.

Let  $S$  denote a non-empty subset of  $\Sigma$  which, without loss of generality, we may assume does not meet the border of  $\Sigma$ . The points  $p$  and  $q$  of  $S$  are said to be connected in  $S$  provided there is a path from  $p$  to  $q$  which is contained in  $S$ . Connectivity is an equivalence relation, and the classes under this relation are called components. Again, the terms 6-connectivity, 26-connectivity, 6-components, and 26-components are utilized depending on the type of path under consideration.

Similarly, we can consider the components of the complement  $\bar{S}$  of  $S$ . Exactly one of these components contains the border of  $\mathcal{L}$ ; this component is called the background of  $S$ . All other components of  $\bar{S}$ , if any, are called cavities in  $S$ . As is the custom in 2D (and 3D) digital geometry, opposite types of connectivity are assumed for  $S$  and  $\bar{S}$  to avoid ambiguous situations. Finally, let  $p$  be a point of  $S$ . We let  $N_{27}(p)$  denote the 27 points in the  $(3 \times 3 \times 3)$  neighborhood of  $p$ , and we let  $N_{125}(p)$  denote the 125 points in the  $(5 \times 5 \times 5)$  neighborhood centered at  $p$ .

#### Surfaces

In [10], the above structure on the 3D-lattice was utilized to introduce the concept of a simple closed surface in providing a non-trivial 3D analog of the 2D Jordan Curve Theorem.

A point  $p \in S$  is a simple surface point provided:

- (i)  $S \cap N_{27}(p)$  has exactly one component adjacent to  $p$  (in the  $S$  sense); denote this component  $A_p$ .
- (ii)  $\bar{S} \cap N_{27}(p)$  has exactly two components,  $C_1$  and  $C_2$ , adjacent to  $p$  (in the  $\bar{S}$  sense).
- (iii) If  $q \in S$  and  $q$  is adjacent to  $p$  (in the  $S$  sense)  $q$  is adjacent (in the  $\bar{S}$  sense) to both  $C_1$  and  $C_2$ .

As observed in [10], there are at most two components of  $\bar{S} \cap N_{125}(p)$  adjacent (in the  $\bar{S}$  sense) to a simple surface point  $p$ . Thus, suppose that  $p$  is a simple surface point of  $S$  and



that each element of  $A_p$  is also a simple surface point of  $S$  (i.e.,  $p$  is not near an "edge"). When  $\bar{S} \cap N_{125}(p)$  has two components adjacent to  $p$ , (the surface at)  $p$  is said to be orientable and  $A_p$  is called a disk. When  $\bar{S} \cap N_{125}(p)$  has only one component adjacent to  $p$ , (the surface at)  $p$  is said to be non-orientable and  $A_p$  is called a cross-cap.

Theorem 0.1 [10] If  $S$  is a connected collection of orientable simple surface points, then  $S$  has exactly one cavity, and  $S$  is said to be a simple closed surface.

Theorem 0.2 [13] There does not exist a 6-connected cross-cap. That is, if  $S$  is a 6-connected subset of  $\Sigma$  and  $p$  is a simple surface point in  $S$  such that each element of  $A_p$  is also a simple surface point of  $S$ , then  $N_{125}(p) \cap \bar{S}$  has two components 26-adjacent to  $p$  and hence  $p$  is orientable.

Thus, the computationally costly assumption of orientability in Theorem 0.1 is unnecessary for 6-connectivity. However, the following example shows that the situation is more complex with respect to 26-connectivity.

Example 0.1 [13] There exists a 26-connected cross-cap. The following set  $S$  (of "1's") is 26-connected, the central point  $p$  in the third plane is a simple surface point of  $S$  and each element of  $A_p$  is also a simple surface point of  $p$ , yet  $N_{125}(p) \cap \bar{S}$  has only one component 6-adjacent to  $p$  and hence  $p$  is not orientable.

1st plane

2nd plane

3rd plane

4th plane

5th plane

0 0 0 0 0

0 0 0 1 0

0 0 0 0 0

0 0 0 1 0

0 0 0 0 0

0 0 0 1 0

0 0 1 0 1

0 0 0 1 1

0 0 1 0 1

0 0 0 1 0

0 0 1 0 0

0 1 0 0 0

0 0 1 0 0

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### 3. 26-connected simple closed surfaces

To show that orientability is not necessary in the characterization of 26-connected simple closed surfaces, let us first outline the proof of Theorem 0.1 given in [10]. Suppose  $S$  is a 26-connected collection of [orientable], simple surface points.

- (1) For each  $p = (x_p, y_p, z_p) \in \bar{S}$ , let  $H_p = \{(x, y, z) \in \Sigma \mid x = x_p, y = y_p, \text{ and } z \geq z_p\}$ , the vertical half-line emanating upward from  $p$ .
- (2) Suppose  $p \in \bar{S}$  and  $\alpha = p_1, p_2, \dots, p_n$  is a connected path in  $S$  along  $H_p$  such that  $p_0$  and  $p_{n+1}$  (the points preceding and following  $p$  along  $H_p$ ) are both in  $\bar{S}$ . Consider  $M = \cup \{N_{27}(x) \mid x \in \alpha\}$ . If  $p_0$  and  $p_{n+1}$  are 26-connected by a path in  $M \cap \bar{S}$ , then we say that  $H_p$  touches  $S$  in  $\alpha$ . If  $p_0$  and  $p_{n+1}$  are not 26-connected by a path in  $M \cap \bar{S}$ , then we say that  $H_p$  crosses  $S$  in  $\alpha$ .
- (3) If  $p \in \bar{S}$ , and  $H_p$  crosses  $S$  an odd number of times, we say that  $p$  is inside  $S$ . If  $p \in \bar{S}$ , and  $H_p$  crosses  $S$  an even number of times, we say that  $p$  is outside  $S$ .
- (4) If  $p, q \in \bar{S}$ , let  $C_{p,q} = \{A_{p,q}^* \mid A_{p,q}^* \text{ is a component of } S \cap (H_p \cup H_q)\}$ .

Proposition 1. If  $p, q \in \bar{S}$ ,  $p$  is 6-adjacent to  $q$ , and  $A_{p,q}^* \in C_{p,q}$ , then  $M \cap \bar{S}$  has two components which are 6-adjacent to each element of  $A_{p,q}^*$ , where  $M = \cup \{N_{27}(x) \mid x \in A_{p,q}^*\}$ .

Proposition 2. If  $p, q \in \bar{S}$ ,  $p$  is 6-adjacent to  $q$ , and  $A_{p,q}^* \in C_{p,q}$ , then  $H_p$  crosses  $S$  an odd number of times in  $A_{p,q}^*$  iff  $H_q$  crosses  $S$  an odd number of times in  $A_{p,q}^*$ .

Proposition 3. If  $p$  is 6-connected to  $q$  in  $\bar{S}$ , then either both are inside of  $S$  or both are outside of  $S$ .

Proposition 4. The inside and outside of  $S$  are both non-empty and  $S$  has exactly one cavity.

A detailed examination of the above proof shows that the use of orientability is restricted to establishing Proposition 1, where it is assumed that for each  $x \in A_{p,q}^*$ ,  $N_{125}(x) \cap \bar{S}$  has two components which are each 6-adjacent to  $x$ . Thus, our goal is to provide a proof of Proposition 1 without such an assumption. As in [13], to establish properties of simple surface points, in which symmetry between the two types of connectivity fails, requires considerable combinatorial detail and explicit notation.

Notation. If  $M \subseteq \Sigma$ , let  $\bar{M} = M \cap \bar{S}$ . For each  $p = (x_p, y_p, z_p)$ , let  $p(i, j, k) = (x_p + i, y_p + j, z_p + k)$ . In addition, let  $p^+$  denote  $p(0, 0, 1)$  and  $p^-$  denote  $p(0, 0, -1)$ .

For  $k$  an integer, let:

- (1)  $N_p^k = \{p(i, j, k) \mid -1 \leq i \leq 1, -1 \leq j \leq 1\}$ ,
- (2)  $N_p^{k=m, n} = \bigcup_{k=m, n} N_p^k$ , and
- (3)  $N_p = N_p^{k=-1, 1}$ .

For example:

$$N_p = N_{27}(p)$$

$$N_p^{-2} = \text{the } 3 \times 3 \text{ plane centered on } p(0,0,-2)$$

$$\bar{N}_p^1 = N_p^1 \cap \bar{S}$$

$$H_p = \{p(0,0,k) \mid k \geq 0\}$$

Finally, if  $p, q \in \bar{S}$ ,  $p$  is 6-adjacent to  $q$ , and  $A_{p,q}^*$  is a component of  $(H_p \cup H_q) \cap S$ , then there exist integers  $n_0$  and  $m$  such that  $A_{p,q}^* = \bigcup_{n=n_0}^m (A_{p,q}^* \cap \{p(0,0,n), q(0,0,n)\})$ .

(1) For  $n_0 \leq n \leq m$ , let  $A_{p,q}^*(n) = A_{p,q}^* \cap \{p(0,0,n), q(0,0,n)\} \neq \emptyset$

(2) For  $n_0 \leq n \leq m$ , let  $B_{p,q}^*(n) = \{p(0,0,n), q(0,0,n)\}$ .

(3)  $B_{p,q}^* = \bigcup_{n=n_0}^m B_{p,q}^*(n)$ .

Our goal is now to establish that if  $\{p, q\} \subseteq \bar{S}$ ,  $p$  is 6-adjacent to  $q$ , and  $A_{p,q}^*$  is a component of  $(H_p \cup H_q) \cap S$ , then  $\bar{M}$ , where  $M = \{N_x \mid x \in B_{p,q}^*\}$ , has two components which are each 6-adjacent to every element of  $A_{p,q}^*$ .

Lemma 1. If  $p$  is a simple surface point of  $S$  and  $x \in \bar{N}_p$ , then  $x$  is 6-connected to  $p$  in  $\bar{N}_p$ .

Proof: Suppose not. Then W.L.G., either (1)  $x \in N_p^0$ , (2)  $x = p(1,1,1)$  and  $p(1,1,0) \in S$ , or (3)  $x = p(0,1,1)$  and  $p(1,1,1) \in S$ .

If (1), then W.L.G. let  $x = p(1,1,0)$ . Now,  $\{p(1,0,0), p(0,1,0)\} \subseteq S$  and either (i)  $\{p(1,1,1), p(1,1,-1)\} \cap S \neq \emptyset$  or (ii)  $\{p(1,1,1), p(1,1,-1)\} \subseteq \bar{S}$ . If (i), then W.L.G. let  $p(1,1,1) \in S$ . But now  $p$  cannot be 6-adjacent to two components of  $\bar{N}_{p(1,1,1)}$ . # If (ii), then  $p(1,0,0)$  cannot be 6-adjacent to two components of  $\bar{N}_p$  which are 6-adjacent to  $p$ . #

If (2), then  $p$  cannot be 6-adjacent to two components of  $\bar{N}_{p(1,1,0)}$ . Hence  $p \in \bar{S}$ , and therefore  $\{p(0,1,1), p(1,0,1)\} \subseteq S$ . Now,  $\bar{N}_{p(0,1,1)}$  has exactly two components,  $C_1$  and  $C_2$ , 6-adjacent to  $p(0,1,1)$ . Since  $p(1,1,1)$  is 6-adjacent to  $p(0,1,1)$ ,  $p(1,1,1)$  must be in one of  $C_1$  and  $C_2$ , say  $C_1$ . Now, since  $p(1,0,0)$  cannot be 6-connected to  $p(0,1,1)$  in  $\bar{N}_{p(0,1,1)}$ , it can be in neither  $C_1$  or  $C_2$ . Thus  $p(0,0,1) \in C_2$  or else  $p(1,0,1)$  could not be 6-adjacent to both  $C_1$  and  $C_2$ . But  $p$  is 6-adjacent to both  $C_1$  and  $C_2$  in  $\bar{N}_{p(0,1,1)}$ . Therefore, either (i)  $p(0,1,0) \in C_1$ , or (ii)  $p(-1,0,0) \in C_1$ . If (i), then  $p(1,2,0) \in C_2$  or else  $p(1,1,0)$  could not be 6-adjacent to  $C_2$ . Thus,  $\{p(0,2,0), p(1,2,1)\} \subseteq S$  or else  $C_1$  would be 6-adjacent to  $C_2$ . However, now there can be no 6-path in  $\bar{N}_{p(0,1,1)}$  from  $p(1,2,0) \in C_2$  to  $p(0,1,1)$ . # If (ii), then  $p(-1,0,1) \in S$ . Hence, since there must be a 6-path in  $\bar{N}_{p(0,1,1)}$  from  $p(-1,0,0)$  to  $p(0,1,1)$ , it follows that  $p(-1,1,0) \in C_1$ . But  $p(0,1,0) \in \bar{S}$  since it cannot be 6-adjacent to two components of  $\bar{N}_{p(-1,0,1)}$ . Thus,  $p(0,1,0) \in C_1$  and we again have case (i). #

If (3), then  $\{p(0,0,1), p(1,0,0)\} \subseteq \bar{S}$ . However, now  $p$  cannot be 6-adjacent to two components of  $\bar{N}_{p(1,1,1)}$ . #

Lemma 2. If  $p, q \in S$  and  $q \in A_p$ , then  $\bar{N}_p \cup \bar{N}_q$  has two components each of which is 6-adjacent to both  $p$  and  $q$ . Equivalently, the two components of  $\bar{N}_p$  are not merged by a 6-path in  $\bar{N}_q$ .

Proof: Suppose not. Let  $C_1$  and  $C_2$  denote the two components of  $\bar{N}_p$ , and let  $C'_1$  and  $C'_2$  denote the two components of  $\bar{N}_q$ . Now, let  $M = N_q \cap N_p$ . There must exist a 6-path  $\alpha$  contained in  $\bar{N}_q/M$  such that  $\alpha$  is 6-adjacent to  $y_1 \in C_1 \cap M$  and to  $y_2 \in C_2 \cap M$ . Furthermore,  $q$  is 6-adjacent to both  $C_1 \cap M$  and  $C_2 \cap M$ . Due to symmetry, we can assume W.L.G. that (1)  $q = p+$ , (2)  $q = p(1,1,0)$ , or (3)  $q = p(1,1,1)$ .

If (1), it follows that  $C'_2 \cap M$  must also be 6-adjacent to  $q$ . Suppose not. Then  $p(0,0,2) \in C'_2$  since  $q$  must be 6-adjacent to  $C'_2$  in  $N_q$ . Thus, W.L.G., either (i)  $p(0,1,1) \in C_1$  and  $p(1,0,1) \in C_2$  or (ii)  $p(0,1,1) \in C_1$  and  $p(0,-1,1) \in C_2$ . If (i), then  $p(1,1,1) \in S$ . But then  $p(1,1,1)$  cannot be adjacent to  $C'_2$  in  $N_q$  without merging  $C'_1$  and  $C'_2$ . # If (ii), then one of  $p(1,0,1)$  and  $p(-1,0,1)$  must be in  $C'_2$  since there must exist a 6-path in  $\bar{N}_q$  from  $p(0,0,2)$  to  $p$ . # Therefore,  $q$  is 6-adjacent to each of  $C_1 \cap M$ ,  $C_2 \cap M$  and  $C'_2 \cap M$ . W.L.G., let  $\{p(0,1,1), p(0,-1,1), p(1,0,1)\}$  contain  $y_1, y_2$  and an element of  $C'_2 \cap M$ . Note that  $\{p(1,1,1), p(1,-1,1)\} \subseteq S$  and that some  $x \in \{p(-1,1,1), p(-1,0,1), p(-1,1,1)\}$  must also be in  $S$ . Now,  $\alpha$  cannot connect  $p(0,1,1)$  and  $p(1,0,1)$  or else  $p(1,1,1)$  could not be 6-adjacent to two components of  $\bar{N}_q$ . Similarly,  $\alpha$  cannot connect  $p(0,-1,1)$  and  $p(1,0,1)$  or else  $p(1,-1,1)$  could not be 6-adjacent to two components of  $\bar{N}_q$ . Thus  $\alpha$  must connect  $p(0,1,1)$  and  $p(0,-1,1)$ . But then  $x$  cannot be 6-connected to  $p(1,0,1)$  in  $\bar{N}_q$ . (# from which (1) follows).

If (2), then W.L.G., either (i)  $p(0,1,0) \in C_1$  and  $p(1,0,0) \in C_2$ , (ii)  $p(0,1,0) \in C_1$  and  $p(1,1,1) \in C_2$ , or (iii)  $p(1,1,-1) \in C_1$ ,  $p(1,1,1) \in C_2$ , and  $\{p(0,1,0), p(1,0,0)\} \subseteq S$ . If (i), then  $p$  is 6-adjacent to only one component of  $\bar{N}_q$ . # If (ii), then  $p(0,1,1) \in S$ . Hence,  $p(1,0,0) \in \bar{S}$  since it can be 6-adjacent to only one component of  $\bar{N}_{p(0,1,1)}$ . However, if  $p(1,0,0) \in C_2$  then we have case (i) again. But if  $p(1,0,0) \in C_1$ ,  $q$  can be adjacent to only one component of  $\bar{N}_p$ . # If (iii), then each point of  $M$  not in  $N_p^0$  must be in  $\bar{S}$  and  $\bar{M}$  has only two components. (# from which (2) follows).

If (3), then W.L.G., either (i)  $p(0,1,0) \in C_1$  and  $p(1,0,1) \in C_2$  or (ii)  $p(0,1,1) \in C_1$  and  $p(1,1,0) \in C_2$ . In either case,  $p$  can be adjacent to only one component of  $\bar{N}_q$ . # This completes the proof.

Lemma 3. If  $p \in S$ ,  $q \in \bar{S}$ , and  $q$  is 6-adjacent to  $p$ , then  $\bar{N}_p \cup \bar{N}_q$  has two components each of which is 6-adjacent to  $p$ .

Proof: Suppose not. W.L.G., let  $q = p+$ , let  $C_1$  and  $C_2$  denote the two components of  $\bar{N}_p$  with  $p+ \in C_1$ , and let  $\alpha$  denote a 6-path in  $\bar{N}_q^2$  from  $y_1 \in N_p^1 \cap C_1$  to  $y_2 \in N_p^1 \cap C_2$ . Then  $y_2 \in \{p(-1,-1,1), p(-1,1,1), p(1,-1,1), p(1,1,1)\}$ . W.L.G., let  $y_2 = p(1,1,1)$  and  $\{p(0,1,1), p(1,0,1)\} \subseteq S$ .

[There is no 6-path in  $\bar{N}_p^1 \cup \bar{N}_p^2$  from  $p+$  to  $y_2$ .] Suppose there is such a path  $\beta$ , where W.L.G.  $\beta$  is minimal. Then  $p(1,1,2) \in \beta$  and one of  $p(1,0,2)$  or  $p(0,1,2)$ , say  $p(1,0,2)$ , must be in  $\beta$  between  $p+$  and  $y_2$ . Observe that  $N_{p(1,0,1)} \cap \beta$  cannot be 6-adjacent



to  $p^+$  or else  $p(0,1,1)$  could not be 6-adjacent to two components of  $\bar{N}_{p(1,0,1)}$ . Hence  $p(0,0,2) \in S$ . Now, since  $\beta$  must be 6-adjacent to  $p^+$ ,  $p(1,-1,2) \in \beta$ ,  $p(0,-1,2) \in \beta$  and  $p(0,-1,1) \in S$ . Furthermore,  $p(1,-1,1) \in \bar{S}$  since it can be adjacent to only one component of  $\bar{N}_{p(0,0,2)}$ . However, now  $\beta$  must be 6-adjacent to  $p^+$  via  $p(-1,0,1)$  which is the only remaining possibility. But then  $p^+$  is 6-adjacent to  $\beta \cap N_{p(0,-1,1)}$  and  $p(1,0,1)$  cannot be 6-adjacent to two components of  $\bar{N}_{p(-1,0,1)}$ . # Hence,  $p^+, y_1$ , and  $y_2$  must belong to three distinct components of  $\bar{N}_p^1$ . Thus (i)  $y_1 \in \{p(1,-1,1), p(-1,1,1)\}$  or (ii)  $y_1 = p(-1,-1,1)$ . If (i), W.L.G. let  $y_1 = p(1,-1,1)$ . Then  $p(0,-1,1) \in S$ . Also, since  $C_1$  and  $C_2$  must be 6-adjacent to  $p$ ,  $p(1,1,0) \in C_2$  and  $p(1,-1,0) \in C_2$ . However, now  $p(1,0,0)$  must be in  $S$  and  $p^+ \in \bar{N}_{p(1,0,0)}$ . But  $p^+$  cannot be 6-connected to  $p(1,0,0)$  in  $\bar{N}_{p(1,0,0)}$ . # If (ii), then  $\{p(-1,0,1), p(0,-1,1)\} \subseteq S$ . Furthermore, since  $p(1,-1,1)$  cannot be 6-adjacent to two components of  $\bar{N}_p$ ,  $p(1,-1,1) \in \bar{S}$ . Now, since each of  $y_1, y_2$ , and  $p(1,-1,1)$  must be 6-connected to  $p$  in  $\bar{N}_p$ ,  $\{p(1,1,0), p(1,-1,0), p(-1,-1,0)\} \subseteq \bar{S}$ . Thus, one of  $x = p(1,0,0)$  or  $x = p(0,-1,0)$  must be in  $S$  or  $C_1$  would be 6-adjacent to  $C_2$  in  $N_p$ . However, in either case, we would have  $p^+ \in \bar{N}_x$  but  $p^+$  cannot be 6-connected to  $x$  in  $\bar{N}_x$ . # The proof is complete.

**Lemma 4.** If  $p$  and  $q = p(1,0,1)$  are in  $S$ ,  $p^+ \in \bar{S}$ , and  $y \in \{p(0,-1,2), p(0,0,2), p(0,1,2)\} \cap \bar{S}$ , then  $y$  is 6-connected to  $p$  via a path in  $\bar{N}_p \cup \bar{N}_p^2$ .

Proof: Suppose not. Then  $y \neq p(0,0,2)$ , hence W.L.G. let  $y = p(0,1,2)$ . Note  $\{p(0,0,2), p(0,1,1)\} \subseteq S$  or else  $y$  would be 6-adjacent to  $\bar{N}_p$  and thus 6-connected to  $p$  in  $\bar{N}_p \cup \bar{N}_p^2$ . Now, since  $y$  must be 6-connected to  $q$  in  $\bar{N}_q$ ,  $p(1,1,2) \in \bar{S}$ . Again,  $p(1,1,1)$  must then be in  $S$ . However, now  $p(1,1,1)$  cannot be 6-adjacent to two components of  $\bar{N}_{p(0,0,2)}$ . #

Lemma 5. Suppose  $p$  and  $q = p(1,0,-1)$  are in  $S$  and  $p(0,0,-1) \in \bar{S}$ . If  $y \in \{p(-1,1,0), p(-1,0,0), p(-1,-1,0)\}$  and  $y$  is 6-connected to  $\bar{N}_p^{-2}$  in  $\bar{N}_p^{i=-2,0}$ , then there is a 6-path  $\alpha$  in  $\bar{N}_p^0 \cup \bar{N}_p^{-1}$  from  $y$  to  $\bar{N}_q$ .

Proof: Suppose not. Note that if  $y = p(-1,0,0)$ , then one of  $p(-1,1,0)$  or  $p(-1,-1,0)$  must be in  $\bar{S}$ , or else  $p(-1,0,-1) \in \bar{S}$  and  $y$  is 6-connected to  $\bar{N}_q$  via  $p(-1,0,-1)$ . Hence, W.L.G., let  $y = p(-1,1,0)$  which implies that  $p(0,1,0) \in S$ . Now, either (1)  $p(0,1,-1) \in \bar{S}$  or (2)  $p(0,1,-1) \in S$ . If (1), then  $p(-1,1,-1) \in S$  or else  $y$  would be 6-connected to  $\bar{N}_q$  via  $p(0,1,-1)$ . Hence, since  $y$  is 6-connected to  $\bar{N}_p^{-2}$  in  $\bar{N}_p^{i=-2,0}$ ,  $p(-1,0,0) \in \bar{S}$  and again  $p(-1,0,-1) \in S$ . Now,  $p(-1,-1,0)$  must be in  $\bar{S}$ , and  $p(-1,-1,-1)$  must also be in  $\bar{S}$ . Thus,  $\{p(0,-1,0), p(0,-1,-1)\} \subseteq S$ . However,  $p(0,0,-1) \in \bar{N}_{p(0,-1,0)}$  but since  $q \in S$ ,  $p(0,0,-1)$  cannot be 6-connected to  $p(0,-1,0)$  in  $\bar{N}_{p(0,-1,0)}$ . # If (2), then  $p(0,0,-1) \in \bar{N}_{p(0,1,0)}$ . Hence, since  $\{p,q\} \subseteq S$ ,  $p(-1,0,-1) \in \bar{S}$  and one of  $p(-1,1,-1)$  and  $p(-1,0,0)$  must also be in  $S$ . But now we have a 6-path in  $\bar{N}_p^0 \cup \bar{N}_p^{-1}$  from  $y$  to  $\bar{N}_q$ . # This completes the proof.

Lemma 6. If  $\{p, q\} \subseteq \bar{S}$ ,  $p$  is 6-adjacent to  $q$ , and  $A_{p,q}^*$  is a component of  $(H_p \cup H_q) \cap S$ , then  $M \cap \bar{S}$ , where  $M = \cup \{N_x \mid x \in B_{p,q}^*\}$ , has two components,  $C_1$  and  $C_2$ , which are each 6-adjacent to every element of  $A_{p,q}^*$ .

Proof: W.L.G.  $q = p(0,0,1)$  or  $q = p(1,0,0)$ . If  $q = p(0,0,1)$ , then the proof follows immediately by induction on Lemma 2. Thus, assume  $q = p(1,0,0)$ . Hence, there exist integers  $n_0$  and  $m$  such

that  $A_{p,q}^* = \bigcup_{i=n_0, m} A_{p,q}^*(i)$ , where for  $n_0 \leq i \leq m$ ,  $A_{p,q}^*(i) \neq \emptyset$ . To

simplify notation, for each  $n_0 \leq i \leq m$ , let  $A_i = \bigcup_{n_0 \leq j \leq i} A_{p,q}^*(j)$ ,

$B_i = \bigcup_{n_0 \leq j \leq i} B_{p,q}^*(j)$ , and  $M_i = \cup \{N_x \mid x \in B_i\}$ . Note that  $B_{p,q}^*(i+1) \subseteq M_i$

for each  $n_0 \leq i < m$ . From Lemma 2 and Lemma 3, it follows immedi-

ately that  $\bar{M}_{n_0}$  has two components which are each 6-adjacent

to each element of  $A_{n_0}$ . We now proceed by induction. Assume

$n_0 \leq n < m$  and  $\bar{M}_n$  has two components,  $C_1$  and  $C_2$ , which are each

6-adjacent to every element of  $A_n$ . [To show:  $\bar{M}_{n+1}$  has two

components which are each 6-adjacent to every element of  $A_{n+1}$ .]

Suppose not, then there must exist a 6-path  $\alpha$  in  $\cup \{\bar{N}_x^1 \mid x \in B_{p,q}^*(n+1)\}$

from  $y_1 \in C_1 \cap (\cup \{\bar{N}_x^0 \mid x \in B_{p,q}^*(n+1)\})$  to  $y_2 \in C_2 \cap (\cup \{\bar{N}_x^0 \mid x \in B_{p,q}^*(n+1)\})$ .

Due to the geometric symmetries involved, we need only consider

the following four cases:

- (1)  $A_{p,q}^*(n) = \{p(0,0,n)\}$ ,  $A_{p,q}^*(n+1) = \{p(0,0,n+1)\}$ ,
- (2)  $A_{p,q}^*(n) = \{p(1,0,n)\}$ ,  $A_{p,q}^*(n+1) = \{p(0,0,n+1)\}$ ,
- (3)  $A_{p,q}^*(n) = \{p(1,0,n)\}$ ,  $A_{p,q}^*(n+1) = \{p(0,0,n+1), p(1,0,n+1)\}$ ,
- (4)  $A_{p,q}^*(n) = \{p(0,0,n), p(1,0,n)\}$ .

(1). ( $A_{p,q}^*(n) = \{p(0,0,n)\}$ ,  $A_{p,q}^*(n+1) = \{p(0,0,n+1)\}$ ). It follows immediately from Lemma 2 that  $\bar{M}$ , where  $M = M_n \cup N_{p(0,0,n+1)}^1$ , has two components,  $C_1'$  and  $C_2'$ , each of which is 6-adjacent to every element of  $A_{n+1}$ . Furthermore,  $C_1 \subseteq C_1'$  and  $C_2 \subseteq C_2'$ . Hence, we can assume that  $\alpha$  is a 6-path, contained in the two rightmost columns of  $\bar{N}_{p(1,0,n+1)}^1$ , which is 6-adjacent to  $y_1 \in C_1'$  and  $y_2 \in C_2'$  where  $\{y_1, y_2\}$  is contained in the union of the rightmost column of  $N_{p(0,0,n+1)}^1$  and the rightmost column of  $N_p^0(1,1,n+1)$ . Note that one of  $y_1$  and  $y_2$  (say  $y_1$ ) cannot be 6-connected to  $\bar{N}_{p(0,0,n+1)}$  via a path in  $\bar{N}_{p(1,0,n+1)} \cap \bar{N}_{p(2,0,n+1)}$ , or else the two components of  $\bar{N}_{p(0,0,n+1)}$  would be merged by the rightmost plane of  $N_{p(1,0,n+1)}$  in contradiction (by symmetry) to Lemma 3. However, each of  $y_1$  and  $y_2$  must be 6-connected to  $p(0,0,n+1)$  by a path in  $\bar{M}$ . Thus, W.L.G. let  $y_1 = p(2,1,n+1)$  which implies  $\{p(2,1,n+1), p(2,1,n+2), p(2,1,n)\} \subseteq \bar{S}$  and  $\{p(1,1,n+1), p(2,0,n+1), p(1,1,n), p(2,0,n)\} \subseteq S$ .

(i) [ $p(1,0,n+1) \in C_2'$ ] Since  $y_1 \in C_1'$  must be 6-connected to  $p(0,0,n)$  in  $\bar{M}$ , let  $k$  denote the greatest integer less than  $n$  such that one of  $p(1,1,k)$  and  $p(2,0,k)$  is in  $\bar{S}$ . Then observe that  $p(1,0,k) \in S$  or else either  $p(1,1,k+1)$  is not 6-adjacent to two components of  $\bar{N}_{p(2,0,k+1)}$  or  $p(2,0,k+1)$  is not 6-adjacent to two components of  $\bar{N}_{p(1,1,k+1)}$ . Also, note that if  $k < i \leq n$ ,  $p(1,0,i) \in \bar{S}$ . To see that this is true, suppose for some  $k < i < n$ ,  $p(1,0,i) \in S$ . Let  $j$  be the greatest such  $i$ , then  $p(1,0,j+1) \in \bar{S}$  and  $\{p(1,1,j+1), p(2,0,j+1), p(1,0,j), p(2,0,j)\} \subseteq S$ . But now  $p(2,0,j)$  cannot be 6-adjacent to two components of  $\bar{N}_{p(1,1,j+1)}$ . # Hence, it follows

that  $p(1,0,k+1) \in \bar{S}$  and  $p(1,0,k+1)$  is 6-connected to  $p(1,0,n+1)$  in  $\bar{M}$ . Now,  $p(1,0,k+1)$  and  $p(2,1,k+1)$  are in opposite components of  $\bar{N}_{p(1,0,k)}$  or else one of  $p(1,1,k+1)$  and  $p(2,0,k+1)$  could not be 6-adjacent to two components of  $\bar{N}_{p(1,0,k)}$ . Thus, since  $p(1,0,k) \in A_n$  and  $N_{p(1,0,k)} \subseteq M$ ,  $p(1,0,k+1)$  and  $p(2,1,k+1)$  are in opposite components of  $\bar{M}$ . Hence, since  $p(1,0,n+1)$  is 6-connected to  $p(1,0,k+1)$  in  $\bar{M}_n$  and  $y_1$  is 6-connected to  $p(2,1,k+1)$  in  $\bar{M}$ , we have  $p(1,0,n+1) \in C_2^1$ .

(ii) (Suppose  $y_2$  is also in the rightmost column of  $N_p^0(1,0,n+1)$ .) Thus  $y_2 = p(2,-1,0)$ . Then since two components of  $\bar{N}_{p(0,0,n+1)}$  cannot be merged by the rightmost plane of  $N_{p(1,0,n+1)}$ , it follows that the rightmost column of  $N_{p(1,0,n+1)}^1 \subseteq \bar{S}$ . Hence  $p(1,0,n+2) \in S$  or else  $p(0,2,n+1)$  could not be 6-adjacent to two components of  $\bar{N}_{p(1,1,n+1)}$ . Furthermore,  $p(1,-1,n+1) \in S$  or else  $p(1,0,n+2)$  could not be 6-adjacent to two components of  $p(2,0,n+1)$ . Now, since  $y_2$  is 6-connected to  $p(0,0,n)$  in  $\bar{M}$ , it again follows as in (i) that  $y_2$  and  $p(1,0,n+1)$  must be in opposite components of  $\bar{M}$ . But  $\{y_1, p(1,0,n+1)\} \subseteq C_2^1$ . #

(iii) (Suppose  $y_2$  is in the rightmost column of  $N_{p(0,0,n+1)}^1$ .) Note that  $p(1,0,n+2) \in S$ , or else  $\{p(1,1,n+2), p(2,0,n+2)\} \subseteq S$  and  $\alpha$  could not connect  $y_1$  to  $y_2$ . Thus  $p(0,1,n+1) \in \bar{S}$  since it cannot be 6-adjacent to two components of  $\bar{N}_{p(1,0,n+2)}$ . Furthermore,  $p(0,1,n+1) \in C_1^1$  since  $p(0,1,n+1)$  and  $p(1,0,n+1)$

must be in different components of  $\bar{N}_p(0,0,n+1)$  or else  $p(1,1,n+1)$  could not be 6-adjacent to two components of  $N_p(0,0,n+1)$ . Finally,  $p(0,1,n+2) \in \bar{S}$  since otherwise  $p(0,1,n+1)$  could not be 6-connected to  $p(1,0,n+2)$  in  $\bar{N}_p(1,0,n+2)$ . Hence,  $p(0,1,n+2) \in C'_1$  and  $y_2 \neq p(1,1,n+2)$ . Thus,  $y_2 = p(1,-1,n+2)$ . Now,  $p(1,-1,n+1) \in \bar{S}$ , or else  $p(1,0,n+1)$  could not be 6-connected to  $p(0,0,n+1)$  in  $\bar{N}_p(0,0,n+1)$ . But since  $p(0,1,n+2) \in C'_1$ ,  $\{p(2,0,n+2), p(2,-1,n+2)\} \subseteq \alpha$ . However, it then follows that  $p(1,0,n+2)$  is 6-adjacent to only one component of  $\bar{N}_p(2,0,n+1)$ . (# from which (1) follows.)

(2). ( $A_{p,q}^*(n) = \{p(1,0,n)\}$ ,  $A_{p,q}^*(n+1) = \{p(0,0,n+1)\}$ ). Consider  $N_p(1,0,n) \subseteq M_n$ . Note  $B_1 \subseteq C_1$  and  $B_2 \subseteq C_2$  where  $B_1$  and  $B_2$  are the two components of  $\bar{N}_p(1,0,n)$ . From Lemma 5, if  $y \in \bar{S}$ ,  $y$  is in the leftmost column of  $N_p^0(0,0,n+1)$ , and  $y$  is 6-connected to  $p(1,0,n)$  in  $M_n$ , then  $y$  is 6-connected to  $p(1,0,n)$  in  $\bar{N}_p^{i=-1,0}(0,0,n+1) \cup \bar{N}_p(1,0,n)$ . Furthermore, from Lemma 2, it follows that  $B_1$  and  $B_2$  cannot be merged in  $\bar{N}_p(0,0,n+1)$ . Thus,  $\bar{M}$ , where  $M = M_n \cup \bar{N}_p^1(0,0,n+1)$ , has two components,  $C'_1$  and  $C'_2$ , each of which is 6-adjacent to every element of  $A_{n+1}$ . Furthermore,  $C_1 \subseteq C'_1$  and  $C_2 \subseteq C'_2$ . Hence, as in (1), we can assume that  $\alpha$  is a 6-path in the two rightmost columns of  $\bar{N}_p^1(1,0,n+1)$  which is 6-adjacent to  $y_1 \in C'_1$  and  $y_2 \in C'_2$  where  $\{y_1, y_2\} \subseteq$  the union of the rightmost column of  $N_p^1(0,0,n+1)$  and the rightmost column of  $N_p^0(1,0,n+1)$ .

However, by geometric symmetry to Lemma 4, if either of  $y_1$  or  $y_2$  is in the righthand column of  $N_{p(0,0,n+1)}^1$ , then it is 6-connected to  $p(1,0,n)$  via a path in  $\bar{N}_{p(1,0,n)} \cup \bar{N}_{p(1,0,n)}^2$ . Thus,  $\bar{N}_{p(1,0,n)}^2$  merges two components of  $\bar{N}_{p(1,0,n)}$  in contradiction to Lemma 3. #

(3).  $(A_{p,q}^*(n) = \{p(1,0,n)\}, A_{p,q}^*(n+1) = \{p(0,0,n+1), p(1,0,n+1)\})$ . As in (2) above,  $\bar{M}$ , where  $M = M_n \cup \bar{N}_{p(0,0,n+1)}^1$ , has two components,  $C_1^i$  and  $C_2^i$ , each of which is 6-adjacent to every element of  $A_{n+1}$ . Hence, again we can assume  $\alpha$  is contained in the two rightmost columns of  $\bar{N}_{p(1,0,n+1)}^1$ . However, we now have two components of  $\bar{N}_{p(1,0,n)}$  merged by a 6-path in  $\bar{N}_{p(1,0,n+1)}$ , which violates Lemma 2. #

(4).  $(A_{p,q}^*(n) = \{p(0,0,n), p(1,0,n)\})$ . W.L.G., assume  $p(1,0,n+1) \in S$ . From either Lemma 2 or Lemma 3, we have that  $\bar{M}$ , where  $M = M_n \cup \bar{N}_{p(0,0,n+1)}^1$ , has two components,  $C_1^i$  and  $C_2^i$ , each of which is 6-adjacent to every element of  $A_{n+1}$ . Hence,  $\alpha$  is contained in the two rightmost columns of  $\bar{N}_{p(1,0,n+1)}^1$ . Again, we then arrive at a contradiction to Lemma 2 as in (3) by consideration of  $N_{p(1,0,n+1)} \cup N_{p(1,0,n)}$ . This completes the proof.

#### 4. Conclusion

Theorem 1. If  $S$  is a connected collection of simple surface points then  $S$  has exactly one cavity, and  $S$  is said to be a simple closed surface.

Hence, we now have the above characterization of simple closed surfaces which holds for both 6-connectivity and 26-connectivity. Furthermore, this characterization is of minimal computational cost in that only the smallest three-dimensional neighborhoods ( $3 \times 3 \times 3$ ) of the respective points need to be examined. This completes the study of [10] and [13].



## References

1. A. Rosenfeld and A. C. Kak, Digital Picture Processing, Academic Press, NY, 1976, Ch. 9: Geometry.
2. A. Rosenfeld, Picture Languages, Academic Press, NY, 1979, Ch. 2: Digital Geometry.
3. S. B. Gray, Local properties of binary images in two and three dimensions, Information International, Inc., Boston, MA, January 1970.
4. C. M. Park and A. Rosenfeld, Connectivity and genus in three dimensions, TR-156, Computer Science Center, University of Maryland, College Park, MD, May 1971.
5. J. P. Mylopoulos and T. Pavlidis, On the topological properties of quantized spaces (I and II), J. ACM 18, 1971, 239-254.
6. G. Tzourakis and J. Mylopoulos, Some results on computational topology, J. ACM 20, 1973, 439-455.
7. C. E. Kim and A. Rosenfeld, Convex digital solids, Proc. PRIP-81, 1981, 175-181.
8. A. Rosenfeld, Three-dimensional digital topology, TR-936, Computer Vision Laboratory, Computer Science Center, University of Maryland, College Park, MD; September 1980, Info. Control, in press.
9. A. Rosenfeld, Some properties of digital curves and surfaces, TR-942, Computer Vision Laboratory, Computer Science Center, University of Maryland, College Park, MD, September 1980.
10. D. G. Morgenthaler and A. Rosenfeld, Surfaces in three-dimensional digital images, TR-940, Computer Vision Laboratory, Computer Science Center, University of Maryland, College Park, MD, September 1980.
11. D. G. Morgenthaler, Three-dimensional digital topology: the genus, TR-980, Computer Vision Laboratory, Computer Science Center, University of Maryland, College Park, MD, November 1980.
12. D. G. Morgenthaler, Three-dimensional single points: serial erosion, parallel thinning, and skeletonization, TR-1005, Computer Vision Laboratory, Computer Science Center, University of Maryland, College Park, MD, February 1981.

13. G. M. Reed and A. Rosenfeld, Recognition of surfaces in three-dimensional digital images, TR-1210, Computer Vision Laboratory, Computer Science Center, University of Maryland, College Park, MD, August 1982.
14. E. Artzy, G. Frieder, and G. T. Herman, The theory, design, implementation and evaluation of a three-dimensional surface detection algorithm, Computer Graphics Image Processing 15, 1981, 1-24.
15. G. T. Herman and D. Webster, Surfaces of organs in discrete three-dimensional space, TR-MIPG 46, Dept. of Computer Science, State University of New York at Buffalo, Amherst, NY, 1980.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <b>This is a continuation of a series of papers on the digital geometry of three-dimensional digital images. In earlier reports, D. Morgenthaler and A. Rosenfeld gave symmetric definitions for simple surface points under the concepts of 6-connectivity and 26-connectivity, and they non-trivially characterized a simple closed surface (i.e., a subset of the image which separates its complement into an "inside" and an "outside") as a connected collection of "orientable" simple surface points. Later, the author and A. Rosenfeld</b>		

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established that the computationally costly assumption of orientability is unnecessary for 6-connectivity by proving that orientability, a local property, is implicitly guaranteed within the  $(3 \times 3 \times 3)$ -neighborhood definition of a 6-connected simple surface point. However, they also showed that no such guarantee exists for 26-connectivity. In this report, the author completes this investigation of simple closed surfaces by showing that orientability is ensured globally by 26-connectivity. Hence, a simple closed surface may be efficiently characterized as a connected collection of simple surface points regardless of the type of connectivity in consideration.

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